

Hong Kong Mathematics Olympiad (2024/25)

Heats – Individual Event

香港數學競賽 (2024/25)

初賽個人項目

INSTRUCTIONS

1. Before the start of the examination, you should follow the announcement to first write your School ID, School name, seat Number and your name, in the appropriate space provided on the answer sheet.
宣布開考前，學生須遵照司儀的指示在答題紙適當位置填寫你的學校編號、學校名稱、座位編號及你的姓名。
2. This paper consists of **TWO** sections, A and B.
本試卷分兩部分，即甲部和乙部。
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in the attached Answer Sheet. Do not write in the margins. Answers written in the margins will not be marked.
本試卷各題均須作答，答案須寫在隨附的答題紙中預留的空位內。不可在邊界以外位置書寫。寫於邊界以外的答案，將不予評閱。
4. Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
除特別指明外，所有答案須以數字的真確值表達並化至最簡。
5. No approximation is accepted.
不接受近似值。
6. Unless otherwise specified, all working **NEED NOT** be shown.
除特別指明外，不須列出所有算式。
7. The diagrams in this paper are not necessarily drawn to scale.
本試卷的附圖不一定依比例繪成。
8. No extra time will be given to candidates for filling in your School ID, School name, seat Number and your name after the ‘Time is up’ announcement.
司儀宣布停筆後，考生不會獲得額外時間填寫你的學校編號、學校名稱、座位編號及你的姓名。

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初賽個人項目

Part A

甲部

1. Let $p = \frac{5}{1 + \frac{5}{1 + \frac{5}{1 + \dots}}}$. Find the value of p .

設 $p = \frac{5}{1 + \frac{5}{1 + \frac{5}{1 + \dots}}}$ ，求 p 的值。

2. In Figure 1, a belt is used to tight 3 identical circles together such that these circles are pairwise tangent to each other. If the radius of each circle is 2 cm, find the length of the belt. (Give the answer in terms of π)

圖一中，一條履帶把 3 個大小相同的圓綑在一起，使得它們都是兩兩互切。若圓的半徑為 2 cm，求該履帶的長度。〔答案須以 π 表示〕

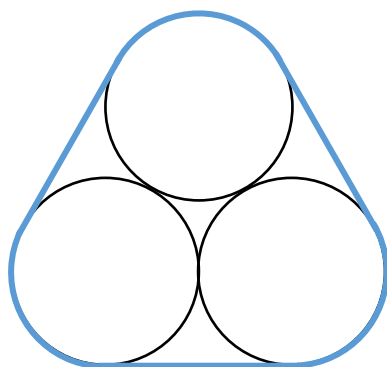


Figure 1

圖一

3. Find the value of $\frac{\tan 1^\circ \times \tan 2^\circ \times \dots \times \tan 89^\circ}{\cos^2 1^\circ + \cos^2 2^\circ + \dots + \cos^2 89^\circ}$.

求 $\frac{\tan 1^\circ \times \tan 2^\circ \times \dots \times \tan 89^\circ}{\cos^2 1^\circ + \cos^2 2^\circ + \dots + \cos^2 89^\circ}$ 的值。

4. In Figure 2, the length of the three sides of a right-angled triangle ABC are $3x$, $3x^2$ and $2x^3$ respectively. Find the value of x .

圖二中，直角三角形 ABC 的三邊長度分別為 $3x$ 、 $3x^2$ 及 $2x^3$ ，求 x 的值。

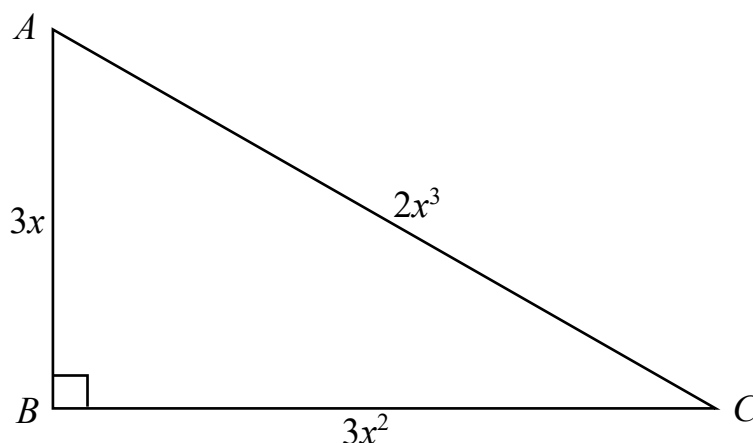


Figure 2

圖二

5. Let x be a real number. Find the minimum value of $x^{2024} - 2024x^{1012} + y$, where y is the largest prime factor of 2024.

設 x 為一實數。求 $x^{2024} - 2024x^{1012} + y$ 的最小值，其中 y 是 2024 的最大質因數。

6. In Figure 3, AB is a diameter of the circle and C is a point on AB , and P lies on the circle such that $PC \perp AB$. Given that $AC = 9$ and $CB = 4$, find the length of PC .

圖三中， AB 是圓的直徑， C 是 AB 上的一點，及 P 是圓上的一點使得 $PC \perp AB$ 。已知 $AC = 9$ 及 $CB = 4$ ，求 PC 的長度。

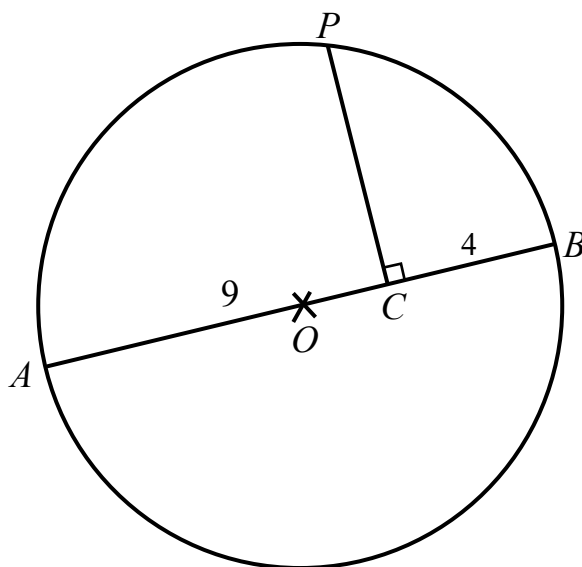


Figure 3

圖三

7. In Figure 4, two circles intersect at points A and B , CD is an external common tangent to the circles and touch the circles at C and D respectively. A straight line passing through A intersects the circles at points E and F respectively, EC and FD intersect at point P . Given that $\angle CBD = 60^\circ$, find $\angle CPD$.

圖四中，兩圓相交於 A 、 B 兩點， CD 為一條外公切線，分別與兩圓相切於點 C 及 D 。過 A 任意做一條直線分別交兩圓於 E 、 F ， EC 交 FD 於點 P 。已知 $\angle CBD = 60^\circ$ ，求 $\angle CPD$ 。

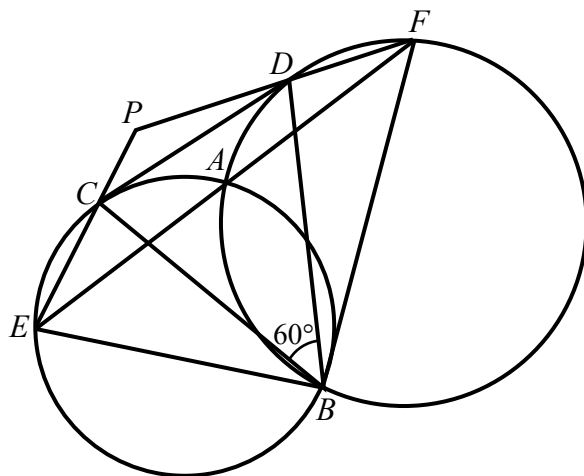


Figure 4

圖四

8. Let x, y be positive integers such that $x(2x+3)(2x+23)=5^y$. Find the value of y .

設 x, y 為正整數使得 $x(2x+3)(2x+23)=5^y$ 。求 y 的值。

9. Given that the equation $(x-a)(x-4)=1$ has 2 integer roots, find the value of a .

已知方程 $(x-a)(x-4)=1$ 有兩個整數根，求 a 的值。

10. Given that x, y are positive integers such that $\frac{1}{x} - \frac{1}{y} = \frac{1}{9}$, find the largest possible value of y .

已知 x, y 為正整數使得 $\frac{1}{x} - \frac{1}{y} = \frac{1}{9}$ ，求 y 的最大可能值。

Part B

乙部

11. Solve the equation $16(9^x) = 72(6^x) - 81(4^x)$.

解方程 $16(9^x) = 72(6^x) - 81(4^x)$ 。

12. Figure 5 shows a triangle ABC , $\angle CAD = 2 \times \angle CBD$, CD is the angle bisector of $\angle ACB$. If $AC = 7$, $AD = 5$, find the length of BC .

圖五所示為一三角形 ABC ， $\angle CAD = 2 \times \angle CBD$ ， CD 是 $\angle ACB$ 的角平分線。若 $AC = 7$ ， $AD = 5$ ，求 BC 的長度。

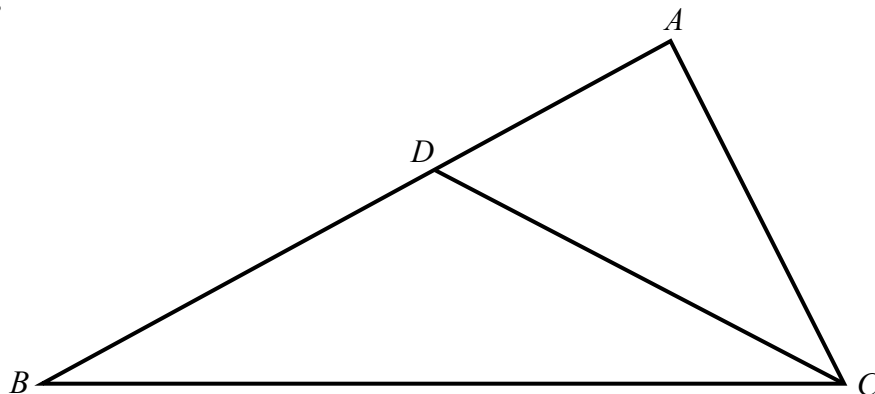


Figure 5

圖五

13. Given that one root of the equation $x^2 - 200x + k = 0$ is 4 less than 50 times the other root of that equation, find the value of k .

已知方程 $x^2 - 200x + k = 0$ 的其中一個根較另一個根的 50 倍少 4，求 k 的值。

14. Let n be a positive integer. Find the least value of n such that $1^2 + 2^2 + 3^2 + \cdots + n^2$ is divisible by 2025.

設 n 為正整數，求 n 的最小值使得 $1^2 + 2^2 + 3^2 + \cdots + n^2$ 可被 2025 整除。

15. Let $y = \frac{x^2 - 2x - 3}{x^2 + 2x + 10}$ and x is a real number. Find the maximum value of y .

設 $y = \frac{x^2 - 2x - 3}{x^2 + 2x + 10}$ 及 x 為實數。求 y 的最大值。

END

完